Statistiques: Série 9
Corrigé

Exercice 1. On a $\mu = 11,5$ et $\sigma = 0,2$.

a) Nouvelles bornes :

$$a = \frac{11,6 - 11,5}{0,2} = 0,5 \text{ et } b = \frac{11,9 - 11,5}{0,2} = 2.$$

Donc

$$P(11,6 \leq X \leq 11,9) = \Phi(2) - \Phi(0,5) = 97,72\% - 69,15\% = 28,57\%.$$

b) Nouvelle borne :

$$b = \frac{11,2 - 11,5}{0,2} = -1,5.$$

Donc

$$P(X \leq 11,2) = \Phi(-1,5)1 - \Phi(1,5) = 1 - 93,32\% = 6,68\%.$$

c) Nouvelle borne :

$$a = \frac{11,4 - 11,5}{0,2} = -0,5.$$

Donc

$$P(X \geq 11,4) = 1 - \Phi(-0,5) = 1 - (1 - \Phi(0,5)) = 1 - 1 + \Phi(0,5) = \Phi(0,5) = 69,15\%.$$

Exercice 2. On a $\mu = 50$ et $\sigma = 1,2$.

a) Nouvelle borne :

$$a = \frac{52 - 50}{1,2} \cong 1,67.$$

Donc

$$P(X \leq 52) = \Phi(1,67) = 95,25\%.$$

Ainsi, $80'000 \cdot 95,25\% = 76'200$ sacs pèseront moins de 52 kg.

b) Nouvelle borne :

$$a = \frac{51 - 50}{1,2} \cong 0,83.$$

Donc

$$P(X \geq 51) = 1 - \Phi(0,83) = 1 - 79,67\% = 20,33\%.$$

Ainsi, $80'000 \cdot 20,33\% = 16'264$ sacs pèseront au moins 51 kg.

c) Nouvelles bornes :

$$a = \frac{48,8-50}{1,2} = -1 \text{ et } b = \frac{51,2-50}{1,2} = 1.$$

Donc

$$\begin{aligned} P(48,8 \leq X \leq 51,2) &= \Phi(1) - \Phi(-1) \\ &= \Phi(1) - (1-\Phi(1)) \\ &= \Phi(1) - 1 + \Phi(1) \\ &= 2 \cdot \Phi(1) - 1 \\ &= 2 \cdot 84,13\% - 1 \\ &= 68,26\%. \end{aligned}$$

Cela représente $80'000 \cdot 68,26\% = 54'608$ sacs.

d) 50'000 sacs représentent

$$\frac{50'000}{80'000} = 62,5\%.$$

On cherche A tel que $P(X \geq A) = 62,5\%$. On a

$$\begin{aligned} P(X \geq A) &= 62,5\% \\ 1 - \Phi(a) &= 62,5\% \\ \Phi(-a) &= 62,5\% \\ -a &= 0,32 \\ a &= -0,32. \end{aligned}$$

On a alors

$$\begin{aligned} a &= \tfrac{A-\mu}{\sigma} \\ -0,32 &= \tfrac{A-50}{1,2} \\ -0,384 &= A - 50 \\ A &= 49,616 \text{ kg}. \end{aligned}$$

Exercice 3. On a $\mu = 100$ et $\sigma = 15$.

a) Nouvelles bornes :

$$a = \frac{100-100}{15} = 0 \text{ et } b = \frac{110-100}{15} \cong 0,67.$$

Donc

$$P(100 \leq X \leq 110) = \Phi(0,67) - \Phi(0) = 74,86\% - 50\% = 24,86\%.$$

b) Nouvelle borne :

$$b = \frac{69-100}{15} \cong -2,07.$$

Donc

$$P(X \leq 69) = \Phi(-2,07) = 1 - \Phi(2,07) = 1 - 98,08\% = 1,92\% < 5\%.$$

c) On cherche B tel que $P(X \geq B) = \frac{1}{3} \cong 33,33\%$. On a

$$\begin{aligned} P(X \leq B) &= 33,33\% \\ \Phi(b) &= 33,33\% \\ 1 - \Phi(-b) &= 33,33\% \\ -\Phi(-b) &= -66,67\% \\ \Phi(-b) &= 66,67\% \\ -b &= 0,43 \\ b &= -0,43. \end{aligned}$$

On a alors

$$\begin{array}{rcl} b & = & \frac{B-\mu}{\sigma} \\ -0,43 & = & \frac{B-100}{15} \\ -6,45 & = & B-100 \\ B & = & 93,55. \end{array}$$

d) On cherche A tel que $P(X \geq A) = 5\%$. On a

$$\begin{array}{rcl} P(X \geq A) & = & 5\% \\ 1-\Phi(a) & = & 5\% \\ -\Phi(a) & = & -95\% \\ \Phi(a) & = & 95\% \\ a & = & 1,65. \end{array}$$

On a alors

$$\begin{array}{rcl} a & = & \frac{A-\mu}{\sigma} \\ 1,65 & = & \frac{A-100}{15} \\ 24,75 & = & A-100 \\ A & = & 124,75. \end{array}$$

Exercice 4. On a $\mu = n \cdot p = 400 \cdot \frac{1}{2} = 200$ et $\sigma = \sqrt{np(1-p)} = \sqrt{400 \cdot \frac{1}{2} \cdot \frac{1}{2}} = 10.$.

a) Nouvelles bornes :

$$a = \frac{190-0,5-200}{10} = -1,05 \text{ et } b = \frac{210+0,5-200}{} = 1,05.$$

Donc

$$\begin{array}{rcl} P(190 \leq X \leq 210) & = & \Phi(1,05) - \Phi(-1,05) \\ & = & \Phi(1,05) - (1-\Phi(1,05)) \\ & = & \Phi(1,05) - 1 + \Phi(1,05) \\ & = & 2 \cdot \Phi(1,05) - 1 \\ & = & 2 \cdot 85,31\% - 1 \\ & = & 70,62\%. \end{array}$$

b) Nouvelle borne :

$$a = \frac{190-0,5-200}{10} = -1,05.$$

Donc

$$P(X \geq 190) = 1 - \Phi(-1,05) = \Phi(1,05) = 85,31\%.$$

Exercice 5. On a $\mu = n \cdot p = 400 \cdot 12\% = 48$ et $\sigma = \sqrt{np(1-p)} = \sqrt{400 \cdot 12\% \cdot 88\%} \cong 6,5.$.

a) Nouvelle borne :

$$b = \frac{30-48+0,5}{6,5} \cong -2,69.$$

Donc

$$P(X \leq 30) = \Phi(-2,69) = 1 - \phi(2,69) = 1 - 99,64\% = 0,36\%.$$

b) Nouvelles bornes :

$$a = \frac{35-48-0,5}{6,5} \cong -2,07 \text{ et } b = \frac{45-48+0,5}{6,5} \cong -0,38.$$

Donc

$$\begin{array}{rcl} P(35 \leq X \leq 45) & = & \Phi(-0,38) - \phi(-2,07) \\ & = & (1 - \Phi(0,38)) - (1 - \Phi(2,07)) \\ & = & 1 - \Phi(0,38) - 1 + \Phi(2,07) \\ & = & \Phi(0,38) + \Phi(2,07) \\ & = & 98,08\% - 64,80\% \\ & = & 33,28\%. \end{array}$$

c) Nouvelle borne :

$$a = \frac{50 - 48 - 0,5}{6,5} \cong 0,23.$$

Donc

$$P(X \geq 50) = 1 - \Phi(0,23) = 1 - 59,10\% = 40,90\%.$$