Statistiques: Série 12
Corrigé

Exercice 1. On a $\mu = 174$ et $\sigma = 7$.

a) Nouvelle borne :
$$a = \frac{185 - 174}{7} \cong 1,57.$$
Sur la table, on trouve $\Phi(1,57) = 94,18\%$. Donc
$$P(X \geq 185) = 1 - 94,18\% = 5,82\%.$$

b) Nouvelle borne :
$$a = \frac{168 - 174}{7} \cong 0,86.$$
Donc
$$P(X \geq 185) = 1-\Phi(-0,85) = 1-(1-(1-\Phi(0,85))) = 1-1+\Phi(0,85) = \Phi(0,85) = 80,51\%.$$

c) Nouvelles bornes :
$$a = \frac{167 - 174}{7} = -1 \text{ et } b = \frac{182 - 174}{7} \cong 1,14.$$
$$\begin{aligned} P(167 \leq X \leq 185) &= \Phi(1,14) - \Phi(-1) \\ &= \Phi(1,14) - (1 - \Phi(1)) \\ &= \Phi(1,14) - 1 + \Phi(1) \\ &= 87,29\% - 1 + 84,13\% \\ &= 71,42\%. \end{aligned}$$

d) Nouvelles bornes :
$$a = \frac{160 - 174}{7} = -2 \text{ et } b = \frac{171 - 174}{7} \cong -0,43.$$
Donc
$$\begin{aligned} P(160 \leq X \leq 171) &= \\ &= \Phi(-0,43) - \Phi(-2) \\ &= (1 - \Phi(0,43)) - (1 - \Phi(2)) \\ &= 1 - \Phi(0,43) - 1 + \Phi(2) \\ &= \Phi(2) - \Phi(0,43) \\ &= 97,72\% - 66,64\% \\ &= 31,08\%. \end{aligned}$$

e) Nouvelles bornes :

$$a = \frac{192 - 174}{7} \cong 2,57 \text{ et } b = \frac{197 - 174}{7} \cong 3,29.$$

Donc

$$P(192 \leq X \leq 197) = \Phi(3,29) - \Phi(2,57) = 99,95\% - 99,42\% = 0,53\%.$$

Exercice 2. On a $\mu = 11,5$ et $\sigma = 0,2$.

a) Nouvelles bornes :

$$a = \frac{11,6 - 11,5}{0,2} = 0,5 \text{ et } b = \frac{11,9 - 11,5}{0,2} = 2.$$

Donc

$$P(11,6 \leq X \leq 11,9) = \Phi(2) - \Phi(0,5) = 97,72\% - 69,15\% = 28,57\%.$$

b) Nouvelle borne :

$$b = \frac{11,2 - 11,5}{0,2} = -1,5.$$

Donc

$$P(X \leq 11,2) = \Phi(-1,5) = 1 - \Phi(1,5) = 1 - 93,32\% = 6,68\%.$$

c) Nouvelle borne :

$$a = \frac{11,4 - 11,5}{0,2} = -0,5.$$

Donc

$$P(X \geq 11,4) = 1 - \Phi(-0,5) = 1 - (1 - \Phi(0,5)) = 1 - 1 + \Phi(0,5) = \Phi(0,5) = 69,15\%.$$

Exercice 3. On a $\mu = 50$ et $\sigma = 1,2$.

a) Nouvelle borne :

$$a = \frac{52 - 50}{1,2} \cong 1,67.$$

Donc

$$P(X \leq 52) = \Phi(1,67) = 95,25\%.$$

Ainsi, $80'000 \cdot 95,25\% = 76'200$ sacs pèseront moins de 52 kg.

b) Nouvelle borne :

$$b = \frac{51 - 50}{1,2} \cong 0,83.$$

Donc

$$P(X \geq 51) = 1 - \Phi(0,83) = 1 - 79,67\% = 20,33\%.$$

Ainsi, $80'000 \cdot 20,33\% = 16'264$ sacs pèseront au moins 51 kg.

c) Nouvelles bornes :

$$a = \frac{48,8-50}{1,2} = -1 \text{ et } b = \frac{51,2-50}{1,2} = 1.$$

Donc

$$\begin{array}{rcl} P(48,8 \leq X \leq 51,2) & = & \Phi(1) - \Phi(-1) \\ & = & \Phi(1) - (1 - \Phi(1)) \\ & = & \Phi(1) - 1 + \Phi(1) \\ & = & 2 \cdot \Phi(1) - 1 \\ & = & 2 \cdot 84,13\% - 1 \\ & = & 68,26\%. \end{array}$$

Cela représente $80'000 \cdot 68,26\% = 54'608$ sacs.

d) 50'000 sacs représentent

$$\frac{50'000}{80'000} = 62,5\%.$$

On cherche A tel que $P(X \geq A) = 62,5\%$. On a

$$\begin{array}{rcl} P(X \geq A) & = & 62,5\% \\ 1 - \Phi(a) & = & 62,5\% \\ \Phi(-a) & = & 62,5\% \\ -a & = & 0,32 \\ a & = & -0,32. \end{array}$$

On a alors

$$\begin{array}{rcl} a & = & \frac{A-\mu}{\sigma} \\ -0,32 & = & \frac{A-50}{1,2} \\ -0,384 & = & A - 50 \\ A & = & 49,616 \text{ kg}. \end{array}$$

Exercice 4. On a $\mu = 100$ et $\sigma = 15$.

a) Nouvelles bornes :

$$a = \frac{100-100}{15} = 0 \text{ et } b = \frac{110-100}{15} \cong 0,67.$$

Donc

$$P(100 \leq X \leq 110) = \Phi(0,67) - \Phi(0) = 74,86\% - 50\% = 24,86\%.$$

b) Nouvelle borne :

$$b = \frac{69-100}{15} \cong -2,07.$$

Donc

$$P(X \leq 69) = \Phi(-2,07) = 1 - \Phi(2,07) = 1 - 98,08\% = 1,92\% < 5\%.$$

c) On cherche B tel que $P(X \geq B) = \frac{1}{3} \cong 33,33\%$. On a

$$\begin{array}{rcl} P(X \leq B) & = & 33,33\% \\ \Phi(b) & = & 33,33\% \\ 1 - \Phi(-b) & = & 33,33\% \\ -\Phi(-b) & = & -66,67\% \\ \Phi(-b) & = & 66,67\% \\ -b & = & 0,43 \\ b & = & -0,43. \end{array}$$

On a alors

$$\begin{array}{rcl} b & = & \frac{B-\mu}{\sigma} \\ -0,43 & = & \frac{B-100}{15} \\ -6,45 & = & B-100 \\ B & = & 93,55. \end{array}$$

d) On cherche A tel que $P(X \geq A) = 5\%$. On a

$$\begin{array}{rcl} P(X \geq A) & = & 5\% \\ 1-\Phi(a) & = & 5\% \\ -\Phi(a) & = & -95\% \\ \Phi(a) & = & 95\% \\ a & = & 1,65. \end{array}$$

On a alors

$$\begin{array}{rcl} a & = & \frac{A-\mu}{\sigma} \\ 1,65 & = & \frac{A-100}{15} \\ 24,75 & = & A-100 \\ A & = & 124,75. \end{array}$$