

Statistiques: Série 8

Corrigé

Exercice 1.

X	$P(\{X\})$
0	1/4
1	1/2
2	1/4

Exercice 2.

X	$P(\{X\})$
1	1/36
2	3/36
3	5/36
4	7/36
5	9/36
6	11/36

$E(X) = 4,47\bar{2}$, $\text{Var}(X) \cong 1,97$ et $\sigma \cong 1,404$.

Exercice 3. Soit X la variable aléatoire qui dénombre le nombre de filles.

a) $P(X = 4) = C_6^4 \cdot \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^2 = 23,4375\%$.

b) $P(X \geq 1) = 1 - P(X = 0) = 1 - C_6^0 \cdot \left(\frac{1}{2}\right)^0 \cdot \left(\frac{1}{2}\right)^6 = 98,4375\%$.

Exercice 4.

a) $P(X = 4) = C_8^4 \cdot \left(\frac{19}{20}\right)^4 \cdot \left(\frac{1}{20}\right)^4 \cong 0,0356\%$.

b) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - C_5^0 \cdot \left(\frac{19}{20}\right)^0 \cdot \left(\frac{1}{20}\right)^5 - C_5^1 \cdot \left(\frac{19}{20}\right)^1 \cdot \left(\frac{1}{20}\right)^4 = 99,997\%$.

Exercice 5.

a) $P(X = 5) = C_8^5 \cdot (60\%)^5 \cdot (40\%)^3 = 27,869\%$.

b) $P(X \geq 1) = 1 - P(X = 0) = 1 - C_5^0 \cdot (60\%)^0 \cdot (40\%)^8 = 99,934\%$.

c) On pose $P(X \geq 1) = 95\%$:

$$\begin{aligned}1 - P(X = 0) &= 95\% \\1 - C_n^0 \cdot (60\%)^0 \cdot (40\%)^n &= 95\% \\1 - (40\%)^n &= 95\% \\-(40\%)^n &= -5\% \\(40\%)^n &= 5\% \\\log((40\%)^n) &= \log(5\%) \\n \cdot \log((40\%)) &= \log(5\%) \\n &= \frac{\log(5\%)}{\log((40\%))} \\n &\cong 3,264.\end{aligned}$$

Ainsi, il devra tirer au moins 4 flèches.

Exercice 6. On pose $P(X \geq 1) = 80\%$:

$$\begin{aligned}1 - P(X = 0) &= 80\% \\1 - C_n^0 \cdot (30\%)^0 \cdot (70\%)^n &= 80\% \\1 - (70\%)^n &= 80\% \\-(70\%)^n &= -20\% \\(70\%)^n &= 20\% \\\log((70\%)^n) &= \log(20\%) \\n \cdot \log(70\%) &= \log(20\%) \\n &= \frac{\log(20\%)}{\log(70\%)} \\n &\cong 4,81.\end{aligned}$$

Il devra donc réaliser au moins 5 placements.