

Statistiques: Série 14

Corrigé

Exercice 1. On a $\mu = n \cdot p = 400 \cdot \frac{1}{2} = 200$ et $\sigma = \sqrt{np(1-p)} = \sqrt{400 \cdot \frac{1}{2} \cdot \frac{1}{2}} = 10$.

a) Nouvelles bornes :

$$a = \frac{190 - 0,5 - 200}{10} = -1,05 \text{ et } b = \frac{210 + 0,5 - 200}{10} = 1,05.$$

Donc

$$\begin{aligned} P(190 \leq X \leq 210) &= \Phi(1,05) - \Phi(-1,05) \\ &= \Phi(1,05) - (1 - \Phi(1,05)) \\ &= \Phi(1,05) - 1 + \Phi(1,05) \\ &= 2 \cdot \Phi(1,05) - 1 \\ &= 2 \cdot 85,31\% - 1 \\ &= 70,62\%. \end{aligned}$$

b) Nouvelle borne :

$$a = \frac{190 - 0,5 - 200}{10} = -1,05.$$

Donc

$$P(X \geq 190) = 1 - \Phi(-1,05) = \Phi(1,05) = 85,31\%.$$

Exercice 2. On a $\mu = n \cdot p = 120 \cdot \frac{1}{6} = 20$ et $\sigma = \sqrt{np(1-p)} = \sqrt{120 \cdot \frac{1}{6} \cdot \frac{5}{6}} \cong 4,08..$

a) Nouvelle borne :

$$a = \frac{18 - 20 - 0,5}{4,08} \cong -0,61.$$

Donc

$$P(X \geq 18) = 1 - \Phi(-0,61) = \Phi(0,61) = 72,91\%.$$

b) Nouvelle borne :

$$b = \frac{25 - 20 + 0,5}{4,08} \cong 1,35.$$

Donc

$$P(X \leq 25) = \Phi(1,35) = 91,15\%.$$

Exercice 3. On a $\mu = n \cdot p = 400 \cdot 12\% = 48$ et $\sigma = \sqrt{np(1-p)} = \sqrt{400 \cdot 12\% \cdot 88\%} \cong 6,5$.

a) Nouvelle borne :

$$b = \frac{30 - 48 + 0,5}{6,5} \cong -2,69.$$

Donc

$$P(X \leq 30) = \Phi(-2,69) = 1 - \phi(2,69) = 1 - 99,64\% = 0,36\%.$$

b) Nouvelles bornes :

$$a = \frac{35 - 48 - 0,5}{6,5} \cong -2,07 \text{ et } b = \frac{45 - 48 + 0,5}{6,5} \cong -0,38.$$

Donc

$$\begin{aligned} P(35 \leq X \leq 45) &= \Phi(-0,38) - \phi(-2,07) \\ &= (1 - \Phi(0,38)) - (1 - \Phi(2,07)) \\ &= 1 - \Phi(0,38) - 1 + \Phi(2,07) \\ &= \Phi(0,38) + \Phi(2,07) \\ &= 98,08\% - 64,80\% \\ &= 33,28\%. \end{aligned}$$

c) Nouvelle borne :

$$a = \frac{50 - 48 - 0,5}{6,5} \cong 0,23.$$

Donc

$$P(X \geq 50) = 1 - \Phi(0,23) = 1 - 59,10\% = 40,90\%.$$